Combining laser light with synchrotron radiation

Part 3

- Frequency Doubling
- Diode laser for spectroscopy

Polarization of the medium

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

In an isotropic medium:

$$\vec{P} = \epsilon_0 \chi \vec{E}$$

$$\vec{D} = \epsilon_0 (1 + \chi) \vec{E} = \epsilon \vec{E}$$

No birefringence!

More general case

For an anisotropic medium:

χ is a tensor

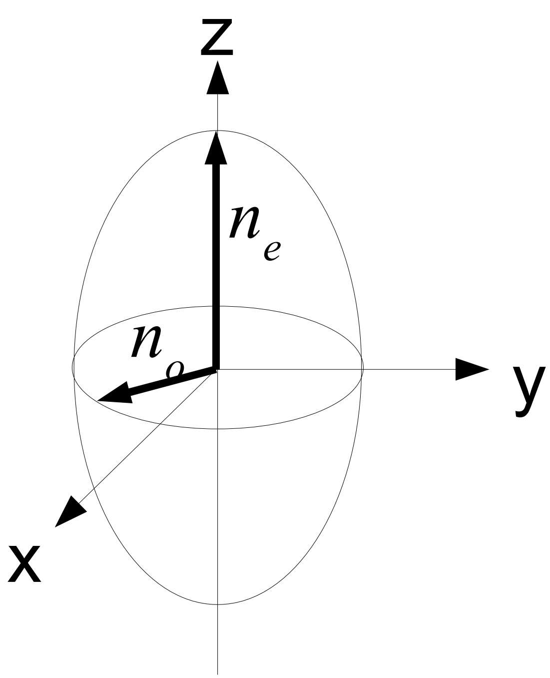
$$P_i = \sum_{j=1,2,3} \epsilon_0 \chi_{ij} E_j$$

Examples: crystals, glass under stress

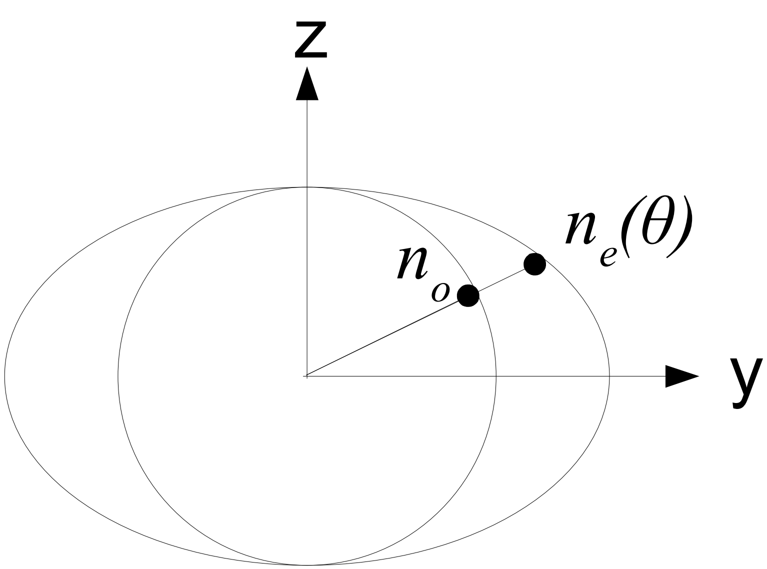
Uniaxial crystals

- One axis with refraction index n_e
- Two axes with refraction index n_o

Index ellipsoid



Normal index surface



The angle between light propagation and crystal axis gives n_e

Frequency doubling

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \epsilon_0 \chi \vec{E} + \underbrace{\vec{e}\left(2\epsilon_0 \chi^{(2)} E^2 + \ldots\right)}_{\vec{P}_{NL}}$$

The quadratic term has double frequency and breaks the point-symmetry:

$$\sin^2(\omega) = \frac{1 + \cos(2\omega)}{2}$$

Maxwell's equations

Gauß' Law

$$\nabla \cdot \vec{D} = \rho$$

No magnetic monopoles

$$\nabla \cdot \vec{B} = 0$$

Faraday's law

$$\nabla \times \vec{E} = -\frac{\delta \vec{B}}{\delta t}$$

Ampère's Law

$$\nabla \times \vec{H} = \vec{J} + \frac{\delta \vec{D}}{\delta t}$$

$$\vec{D} = \epsilon \vec{E} + \vec{P}_{NL} \qquad \vec{J} = \sigma_S \vec{E} \qquad \vec{B} = \mu_0 \vec{H}$$

wave equation

$$\nabla^2 \vec{E} - \frac{\sigma_s}{\epsilon c^2}\frac{\delta \vec{E}}{\delta t} - \frac{1}{c^2}\frac{\delta^2 \vec{E}}{\delta t^2} = \frac{1}{\epsilon c^2}\frac{\delta^2 \vec{P}_{NL}}{\delta t^2}$$

Electromagnetic wave

Driven by second order polarization

Example

An electromagnetic wave in z-direction

$$E_{\omega}(z,t)=\frac{1}{2}\{E(z,\omega)\exp[i(\omega t-k_{\omega}z)]+c.c.\}$$

generates a polarization wave with double frequency.

$$P_{NL}^{2\omega}=\frac{\epsilon_0\chi^{(2)}}{2}\{E^2(z,\omega)\exp[i(2\omega t-2k_{\omega}z)]+c.c.\}$$

The polarization drives an electromagnetic wave

$$E_{2\omega}(z,t)=\frac{1}{2}\{E(z,2\omega)\exp[i(2\omega t-k_{2\omega}z)]+c.c.\}$$

$$k_{\omega}=\frac{n_{\omega}\omega}{c} \qquad k_{2\omega}=\frac{2n_{2\omega}\omega}{c}$$

Phase matching

The second harmonic wave can only
be effectively excited if their velocities match:

Polarization: $$v_P = \frac{2\omega}{2k_\omega} = \frac{\omega}{k_\omega} = v_\omega$$

Electromagnetic wave: $$v_E = \frac{2\omega}{k_{2\omega}}$$

Only possible if fundamental and harmonic wave
don't have parallel polarization

In the photon picture this gives conservation
of momentum: $$\hbar k_{2\omega} = 2\hbar k_\omega$$

More general case

Polarization **not** in plane with the field

$$\vec{E}^{\omega}(\vec{r},t)=\frac{1}{2}\left[\vec{E}^{\omega}(\vec{r},\omega)\,e^{i\omega t}+c.c.\right]$$

$$\vec{P}_{NL}^{2\omega}(\vec{r},t)=\frac{1}{2}\left[\vec{P}^{2\omega}(\vec{r},2\omega)\,e^{2i\omega t}+c.c.\right]$$

$$P_i^{2\omega}=\sum_{j,k=1,2,3}\epsilon_0\,d_{i,j,k}^{2\omega}\,E_j^{\omega}\,E_k^{\omega}$$

Example: KDP

KDP: KH_2PO_4

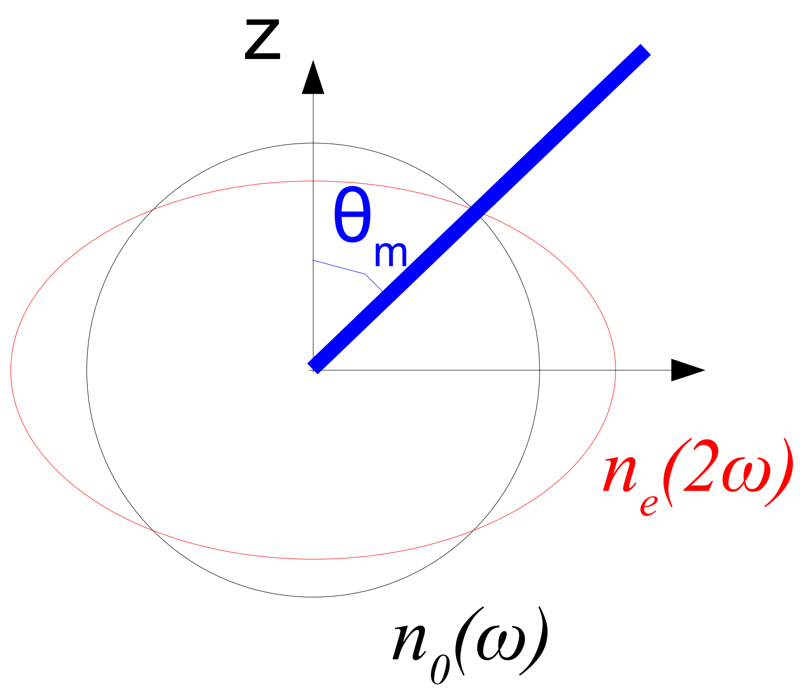
$$P_x = 2\epsilon_0 d_{36} E_y E_z$$

$$P_y = 2\epsilon_0 d_{36} E_z E_x$$

$$P_z = 2\epsilon_0 d_{36} E_x E_y$$

Birefringence can be used to match the velocities

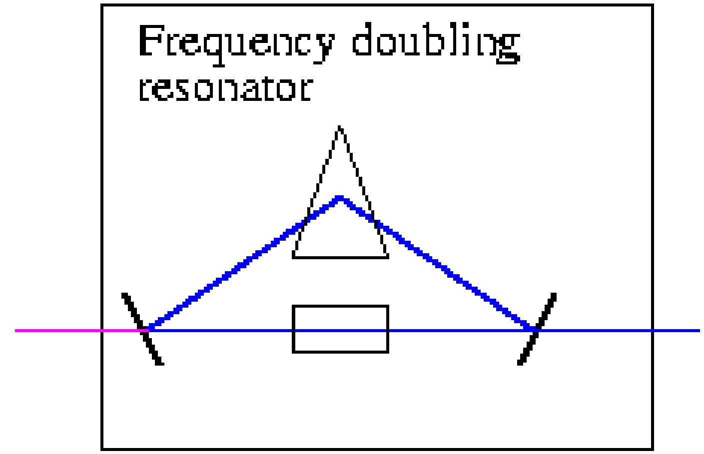
The phase matching angle



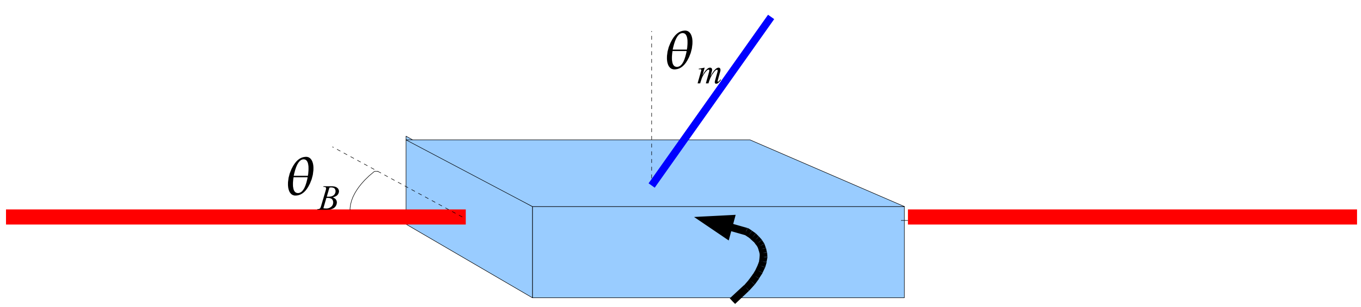
Select matching angle θ_m

$$n_e(2\omega, \theta_m) = n_0(\omega)$$

A commercial doubler system



Crystal cut in Brewser's angle to match the phase



and can be rotated to tune the wavelength

Tuning ranges of a doubler crystals

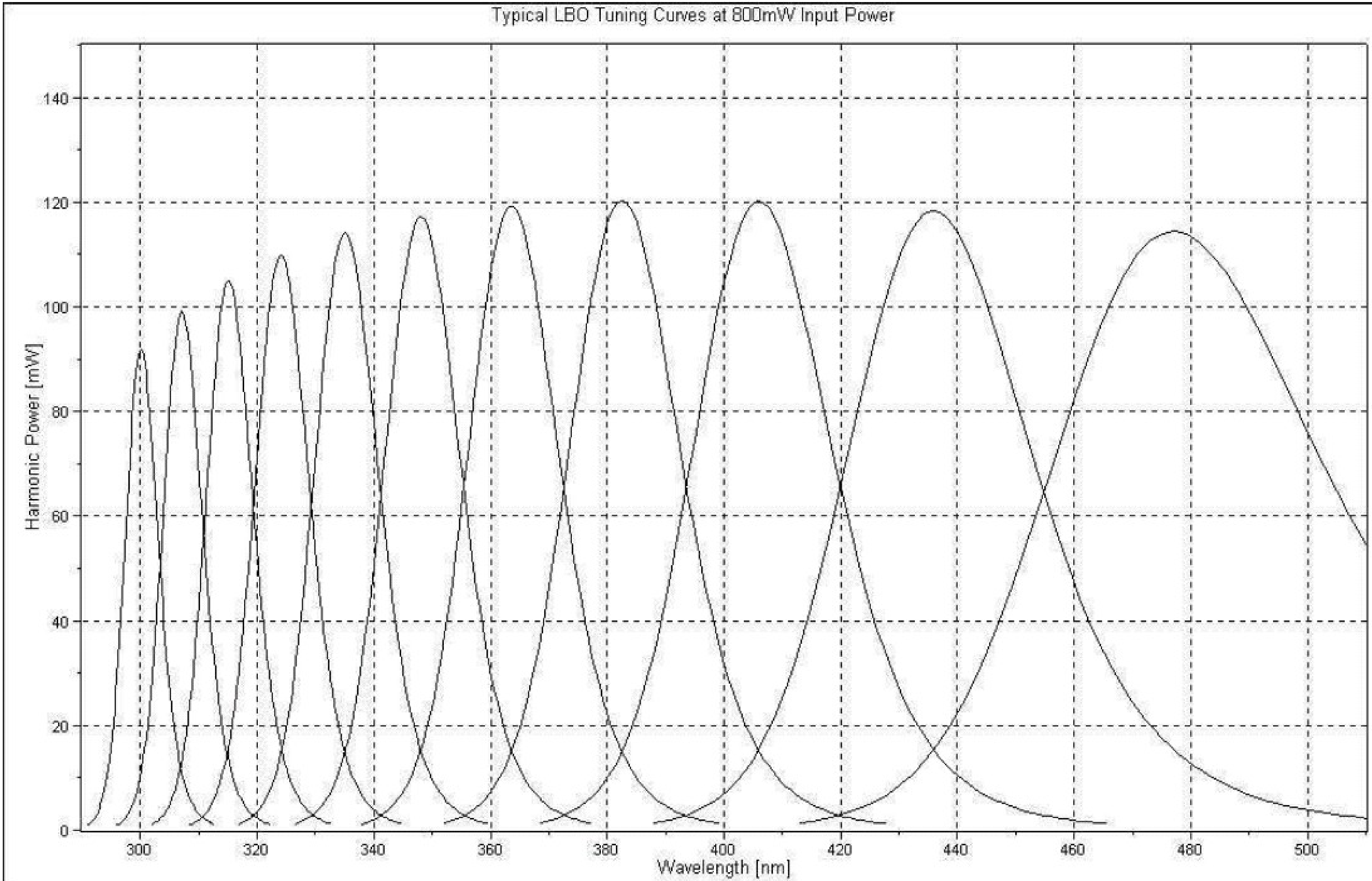
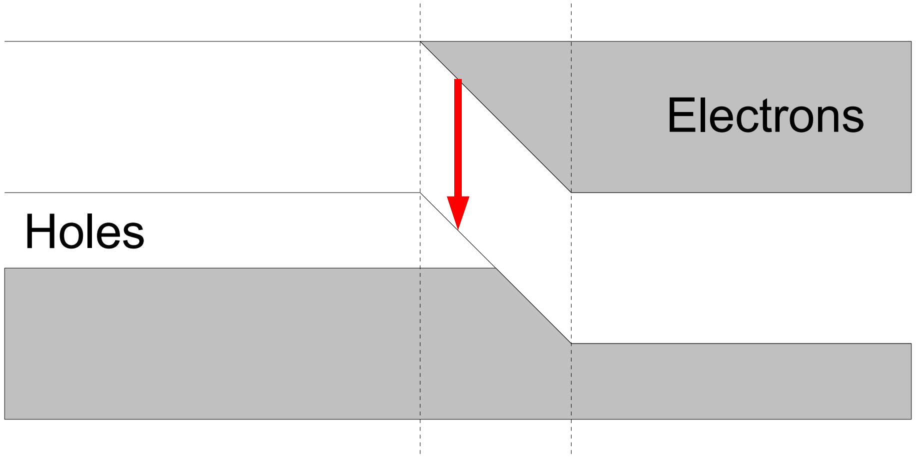


Figure 6-5: Typical LBO Tuning Curves @800mW Input Power

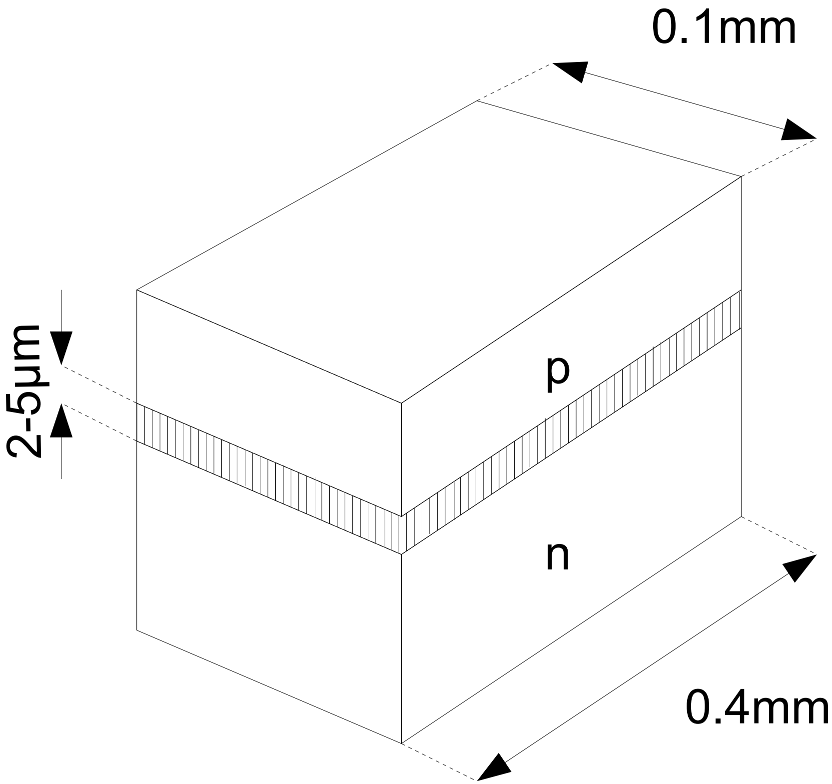
Semiconductor lasers

p-doted



n-doted

Semiconductor lasers



Tuning over Temperature changes refraction index and cavity length

$$\Delta\nu = \frac{\delta\nu}{\delta n}\frac{dn}{dT}\Delta T + \frac{\delta\nu}{\delta L}\frac{dL}{dT}\Delta T$$

$$\Delta\nu = -\nu\left(\frac{1}{n}\frac{dn}{dt} + \frac{1}{L}\frac{dL}{dT}\right)\Delta T$$

But the band gap also changes with temperature

Disadvantages of laser diodes for spectroscopy

Cavity length and and gain profile not independent

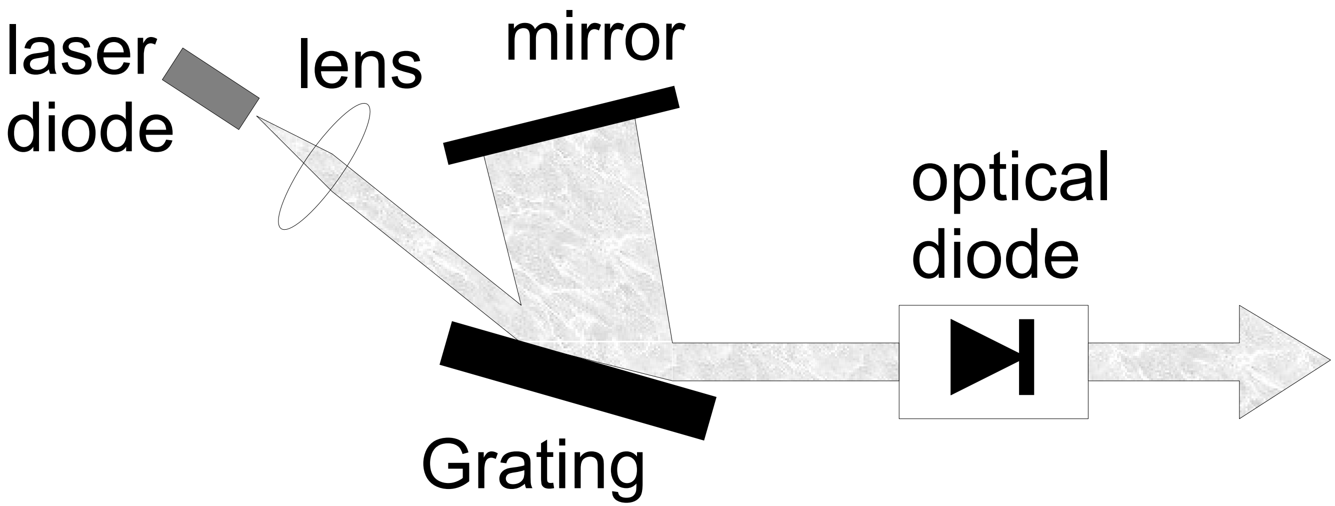
→mode hops during temperature tuning

Very flat active region

→diffraction gives a bad beam profile

Solution: external cavity

Littmann-configuration



Temperature controls gain profile

Mirror angle tunes the cavity

A commercial system

Modified Littrow-configuration (Sacher Lasertechnik)