Combining laser light with synchrotron radiation

Part 4

Wavelength ↔ time structure

More literature:

Amnon Yariv, "Quantum Electronics", 3rd edition
John Wiley & Sons, New York (1988)

Maxwell's equations

Gauß' Law

$$\nabla \cdot \vec{D} = \rho$$

No magnetic monopoles

$$\nabla \cdot \vec{B} = 0$$

Faraday's law

$$\nabla \times \vec{E} = -\frac{\delta \vec{B}}{\delta t}$$

Ampère's Law

$$\nabla \times \vec{H} = \vec{J} + \frac{\delta \vec{D}}{\delta t}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E}$$

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right) = \mu \vec{H}$$

Wave equations

$$\nabla^2 \vec{E} = \epsilon \mu \frac{\delta^2 \vec{E}}{\delta t^2} \qquad \nabla^2 \vec{H} = \epsilon \mu \frac{\delta^2 \vec{H}}{\delta t^2}$$

Free Wave:

$$\vec{E} = \sum_i \vec{A}_i e^{i(\omega_i t + \vec{k}_i \cdot \vec{r})} + c.c.$$

Speed of light:

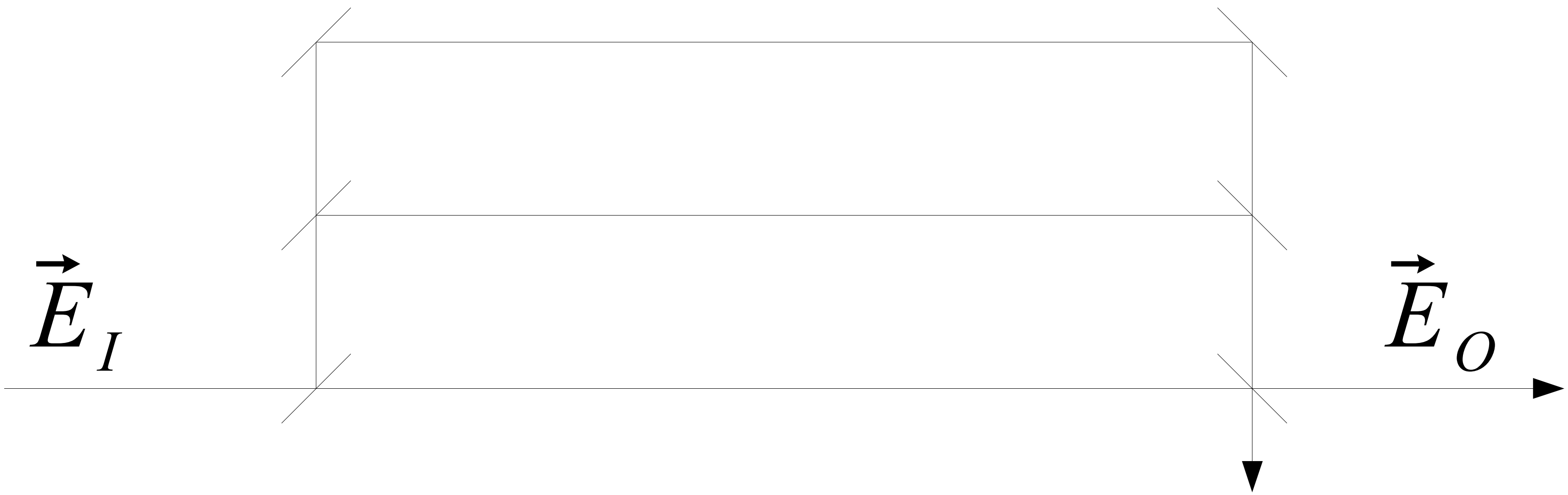
$$v = \frac{\omega_i}{k_i} = \frac{1}{\sqrt{\epsilon \mu}} = \frac{c}{n}$$

Transversal:

$$\vec{E} \cdot \vec{k} = 0$$

Interferometers

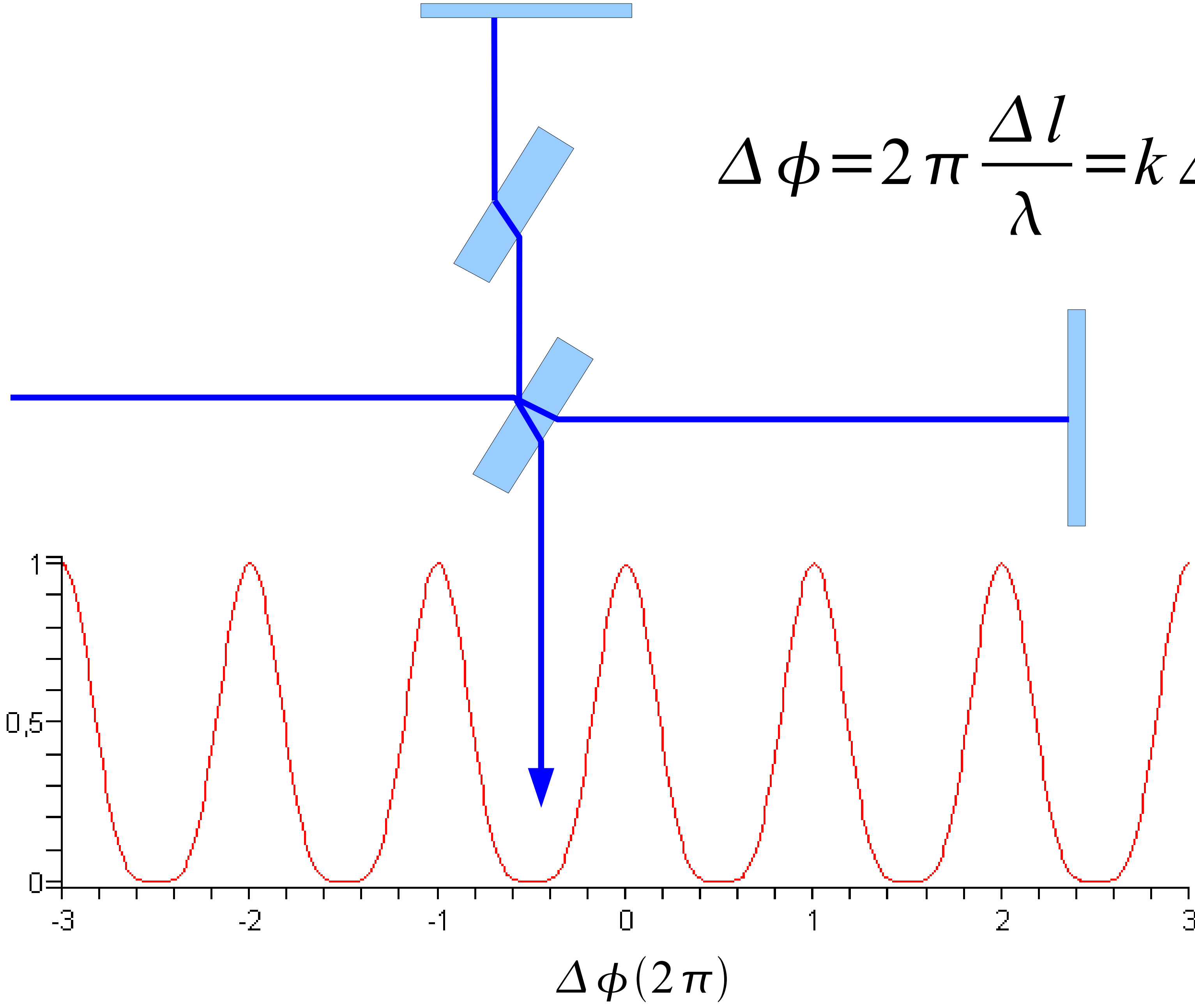
$$\vec{E}_I = \vec{A}\sin(\omega t + \vec{k}\,x)$$



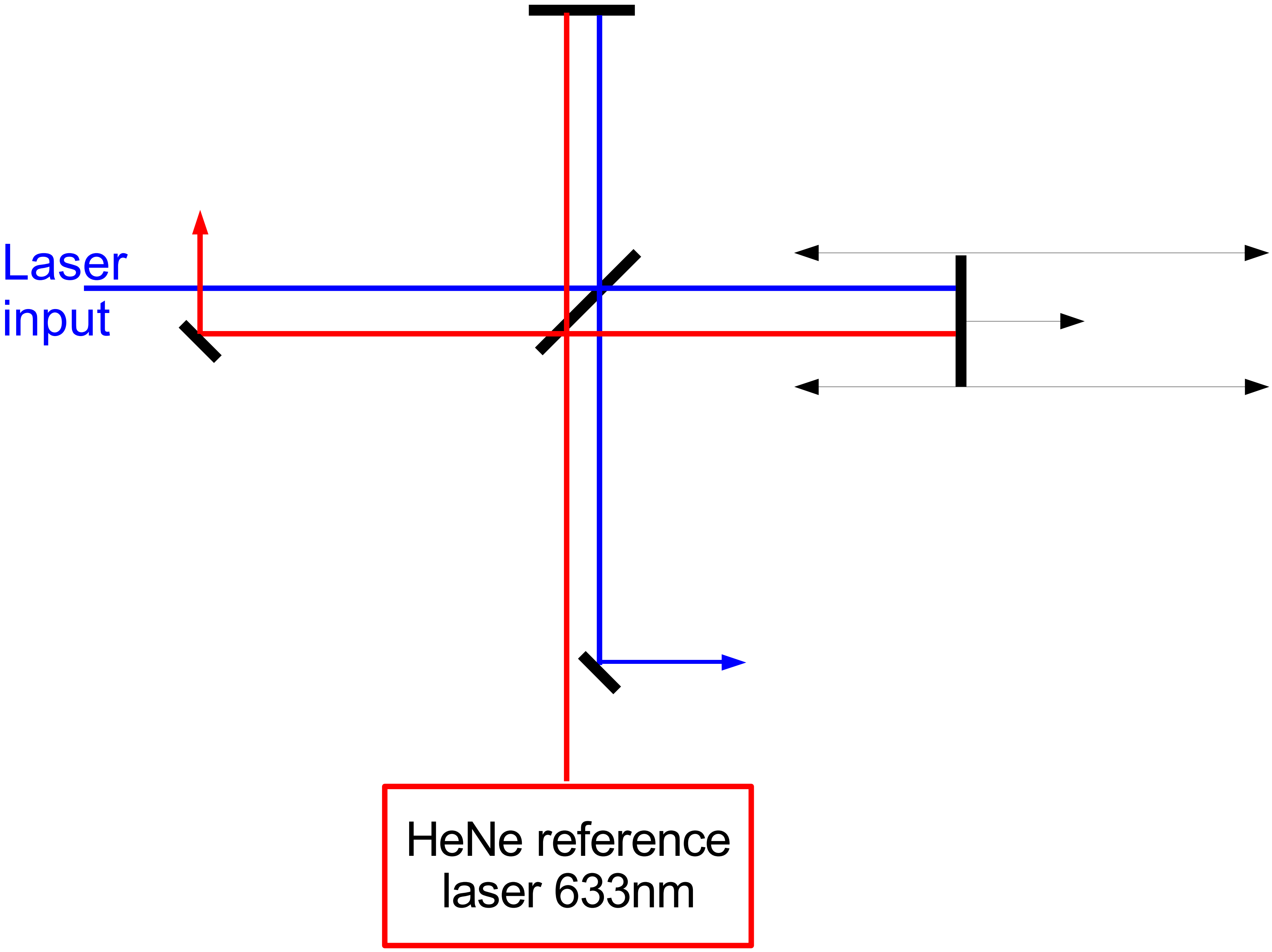
$$\vec{E}_O = \vec{A}_1 \sin(\omega t + \vec{k}\,x + \phi_1) + \vec{A}_2 \sin(\omega t + \vec{k}\,x + \phi_2) + \ldots$$

Michelson Interferometer

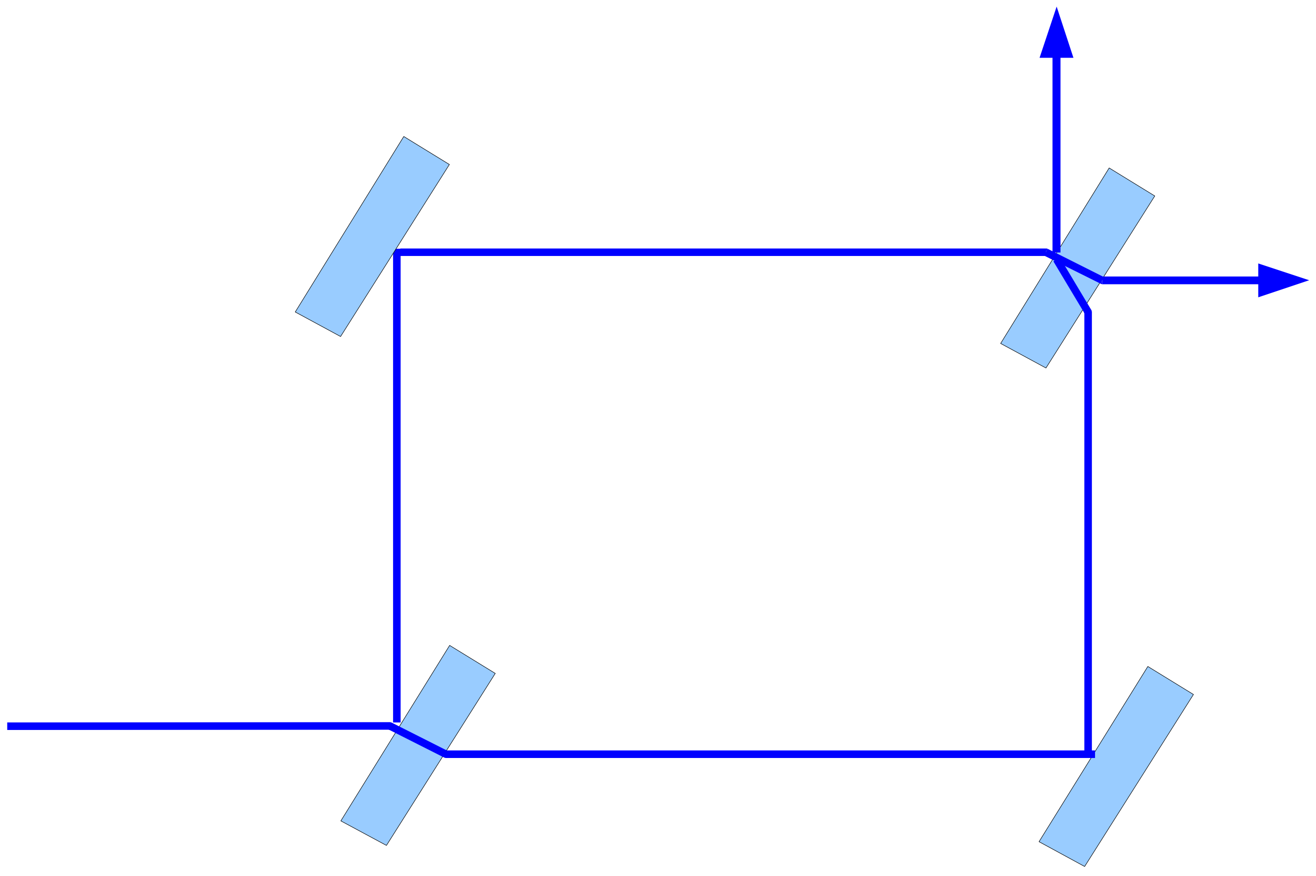
$$\Delta\phi = 2\pi\frac{\Delta l}{\lambda} = k\,\Delta l$$



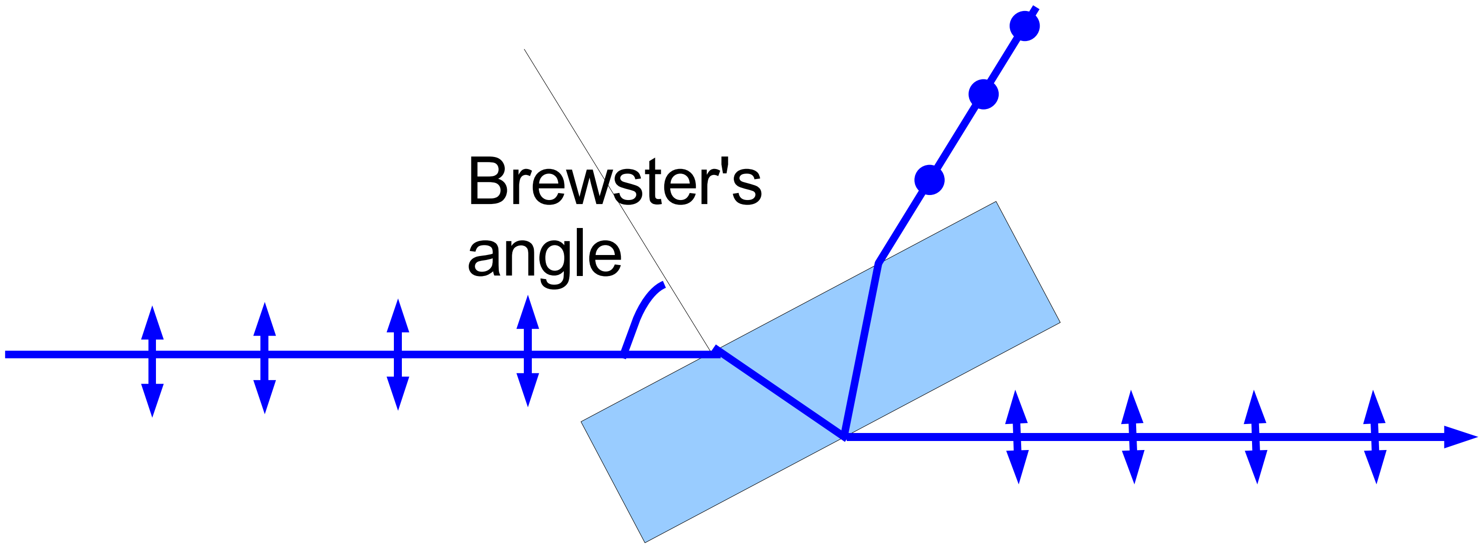
Michelson Lambda-meter



Mach-Zehender Interferometer



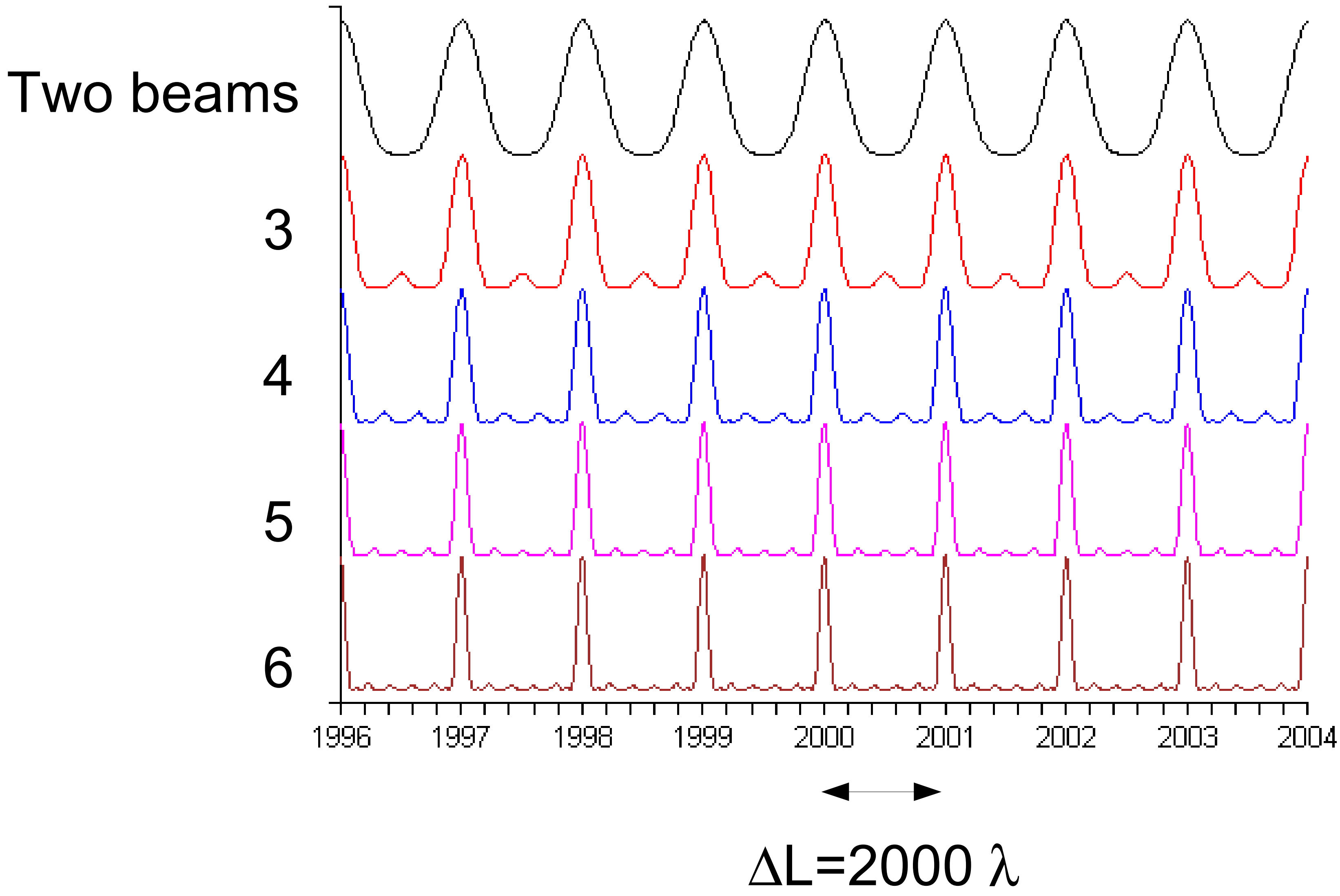
Lyot Filter
(birefringent filter)



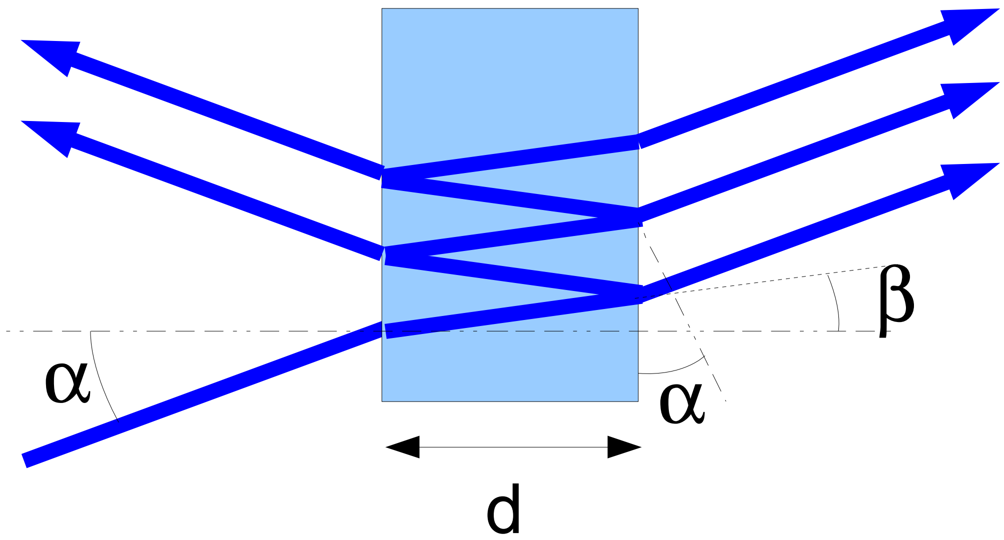
Ordinary and extraordinary beam only recombine to the same linear polarization, if the difference of their optical path lengths is $k\lambda$ with $k=0, \pm 1, \pm 2, ...$

$$n_o\, d - n_e\, d = k\,\lambda$$

Multi beam interference



Fabry-Perot Interferometer

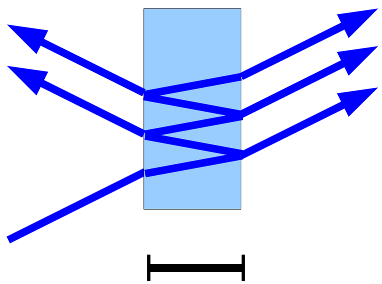


$$\Delta L = \frac{2nd}{\cos\beta} - 2d\tan\beta\sin\alpha$$

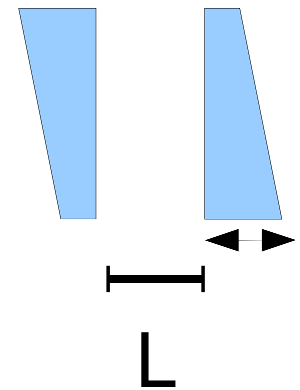
$$\Delta L = 2nd\cos\beta = 2d\sqrt{n^2 - \sin^2\alpha}$$

Fabry-Perot Interferometers

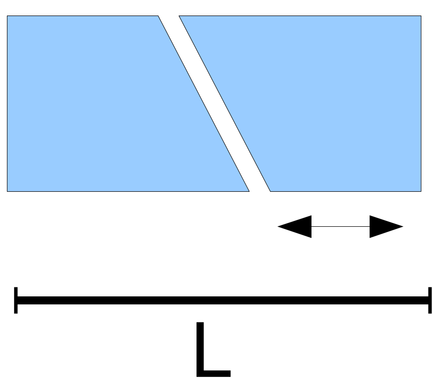
Plane parallel plate



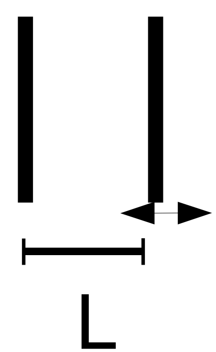
Two glass plates



Glass plate with gap

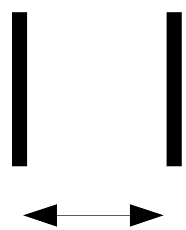


Two mirrors (surfaces with reflection coating)



Example

A 1cm thick Fabry-Perot interferometer (Etalon)



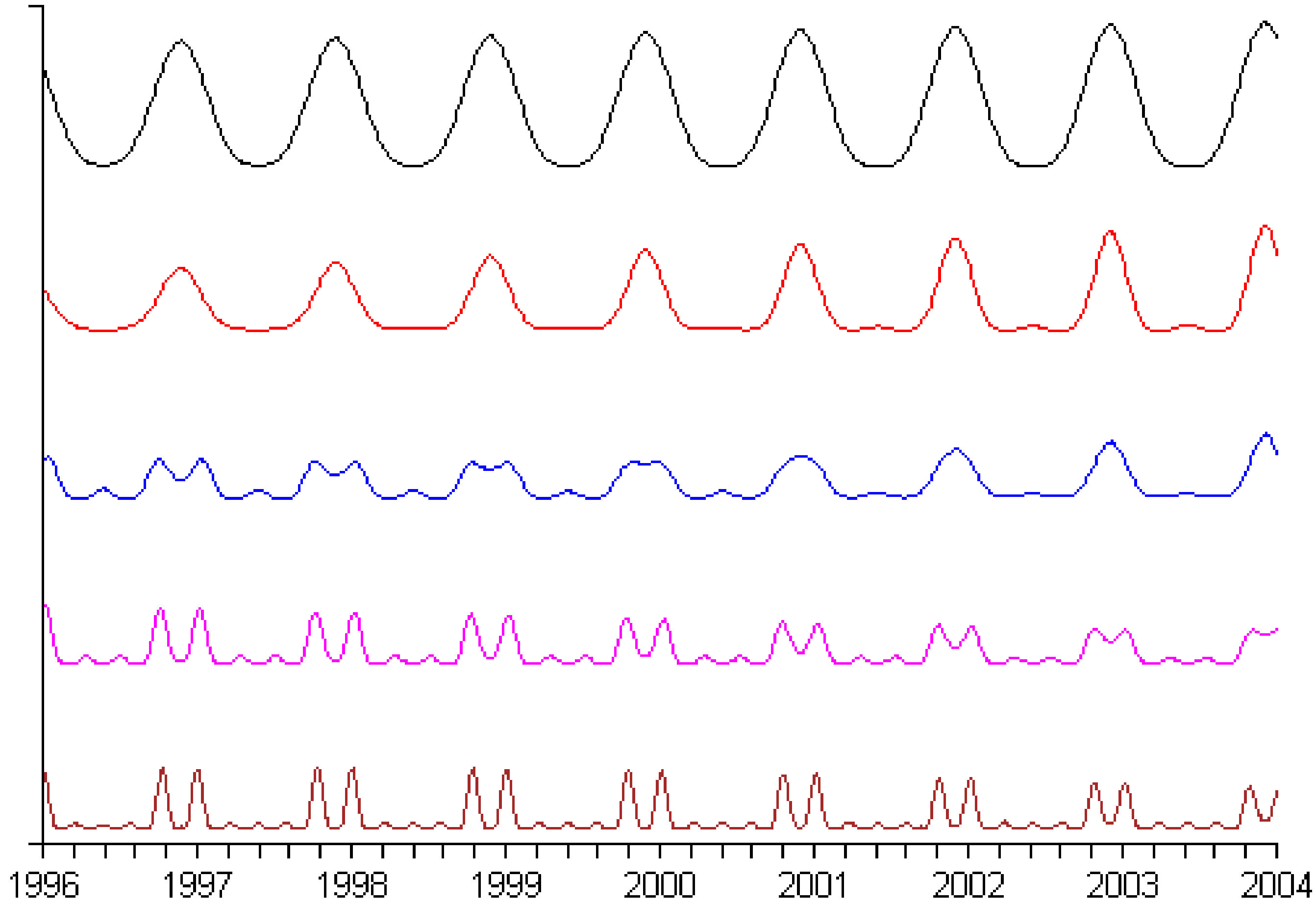
ΔL=2cm

Fits 25000 waves of 800.000 nm
..or 25001 waves of 799.968 nm

374740.57 GHz
374755.56 GHz
-14.99 GHz

Free Spectral Range (FSR): ~15 GHz

Two Waves: $k_2 = 1.0001\, k_1$



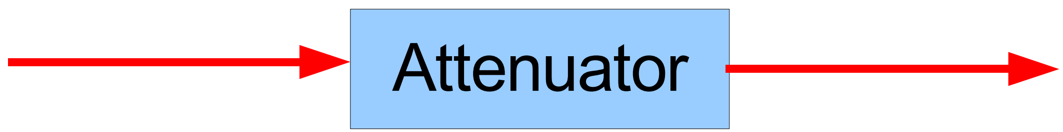
Example: ΔL= 1.6 mm: FSR= 93.7 GHz, 3.125 cm^{-1}

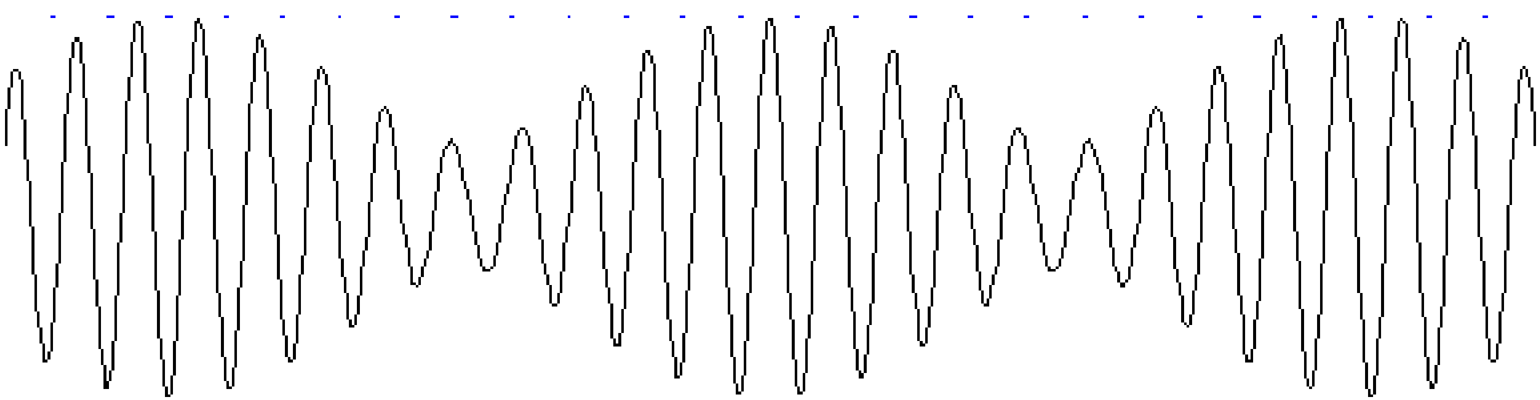
λ_1=800.00 nm $\Rightarrow k_1$=2p 12500 cm^{-1} $\Rightarrow$ 374740 GHz

k_2=2p 12501.25 cm^{-1} $\Rightarrow \lambda_2$=799.92 nm $\Rightarrow$ 374778 GHz

Amplitude modulation

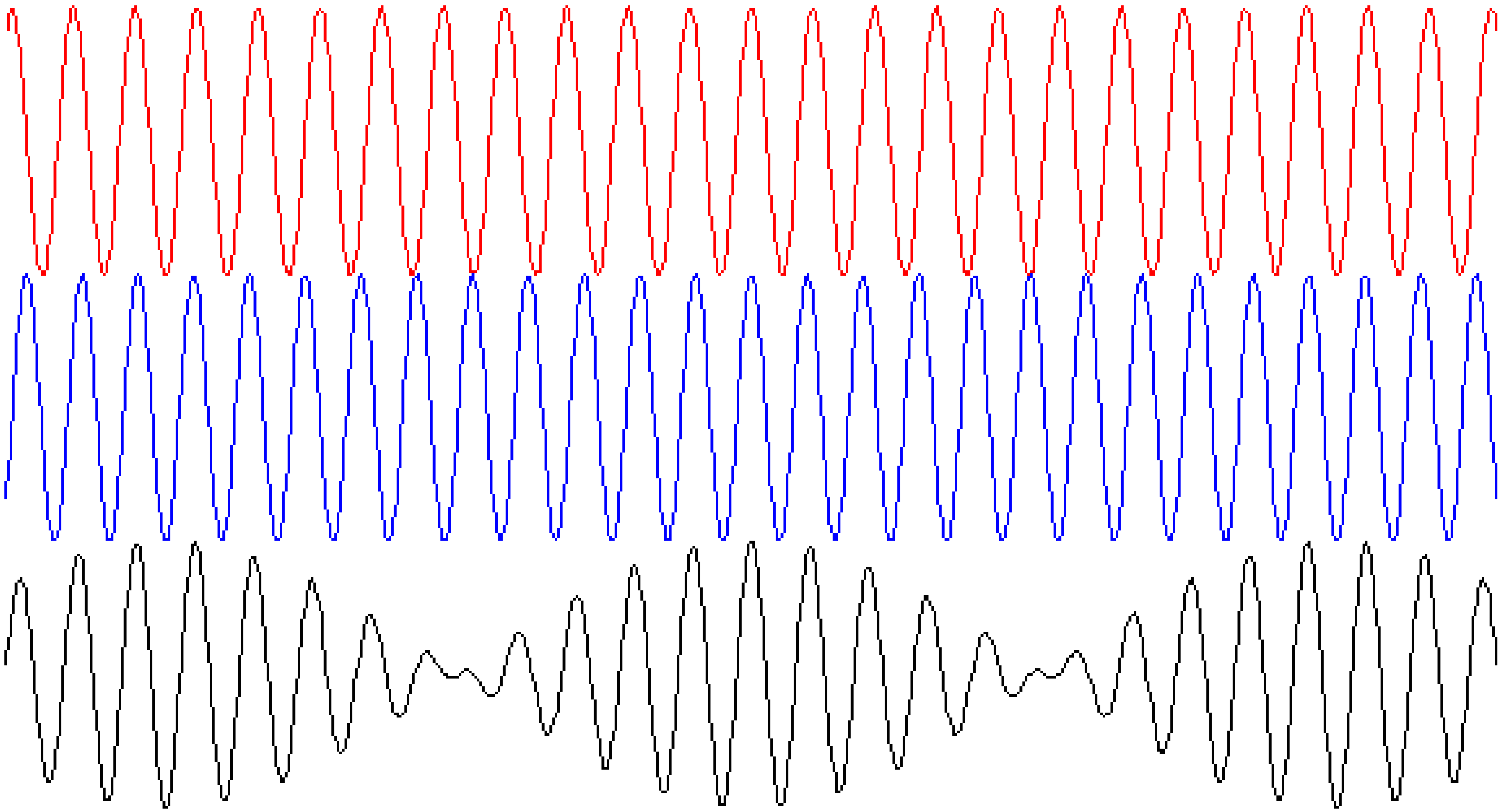
$$\vec{E} = \vec{A}(t, x)\cos(\omega t + k x) \qquad \frac{\omega}{k} = c$$





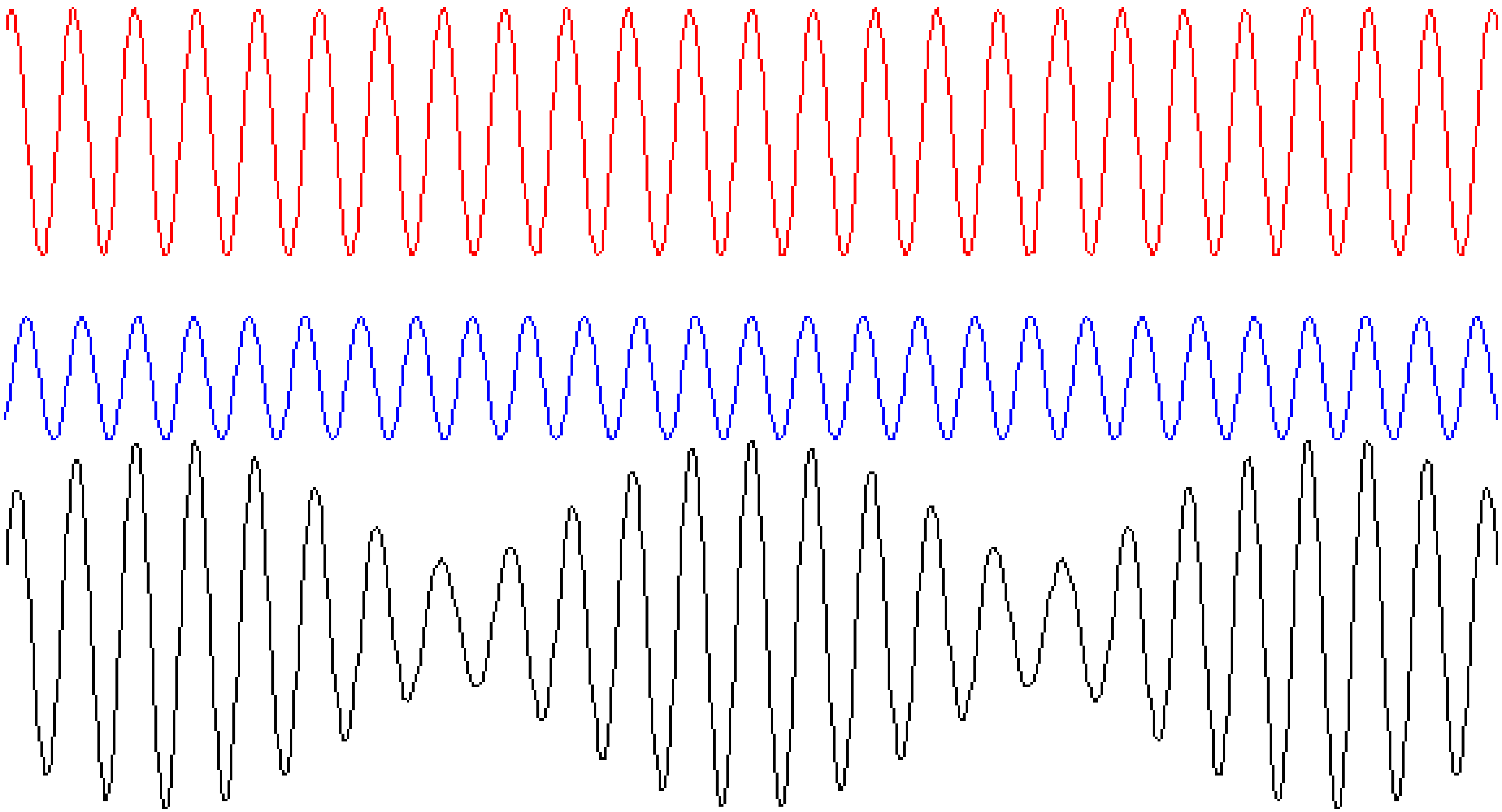
Amplitude modulation

$$\vec{E} = 0.5\,\vec{A}_0 \cos\left((\omega + \Omega)t + (k + K)x\right)$$
$$+ 0.5\,\vec{A}_0 \cos\left((\omega - \Omega)t + (k - K)x\right)$$



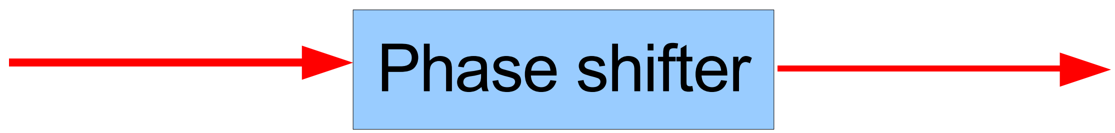
$$\vec{E} = \vec{A}_0 \cos(\Omega\, t + K\, x) \cos(\omega\, t + k\, x)$$

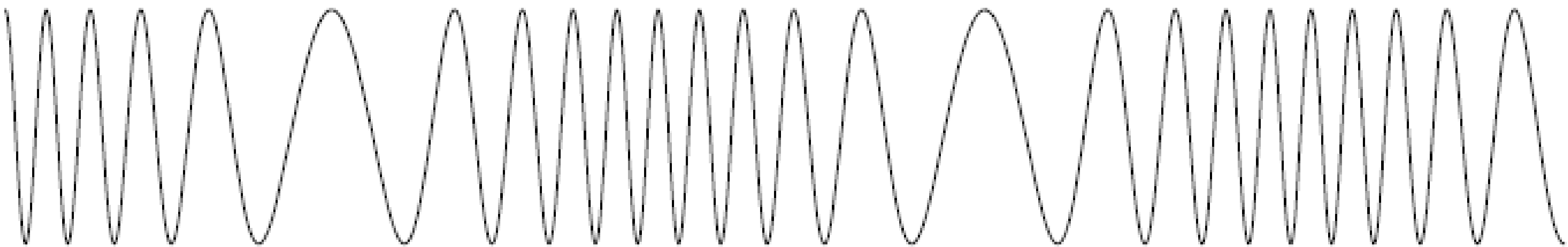
Amplitude modulation



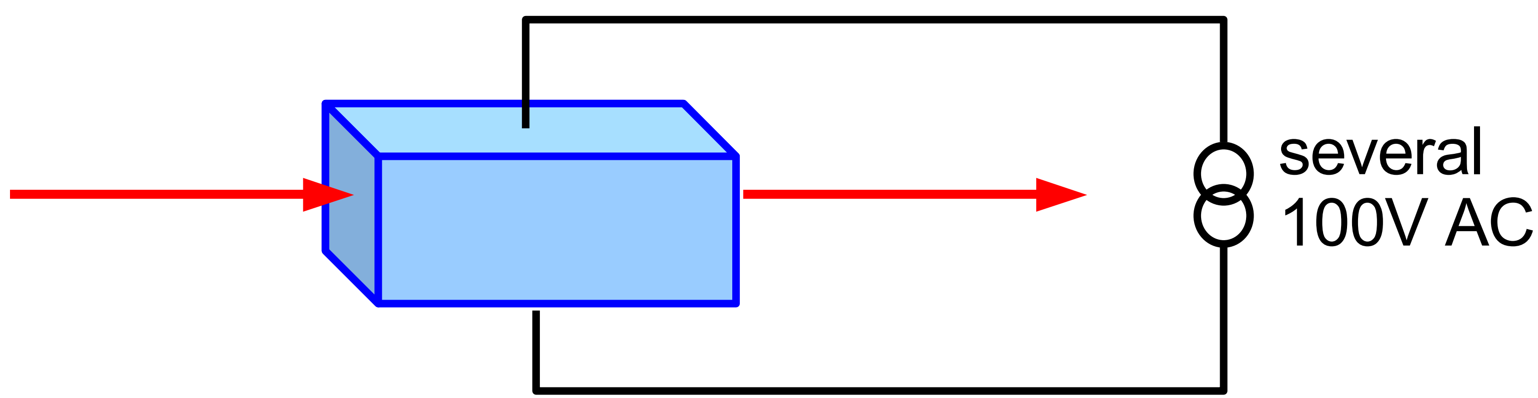
Phase modulation

$$\vec{E}(t)=\vec{A}\cos(\omega t+M\sin\Omega t) \qquad \frac{\omega}{k}=c$$





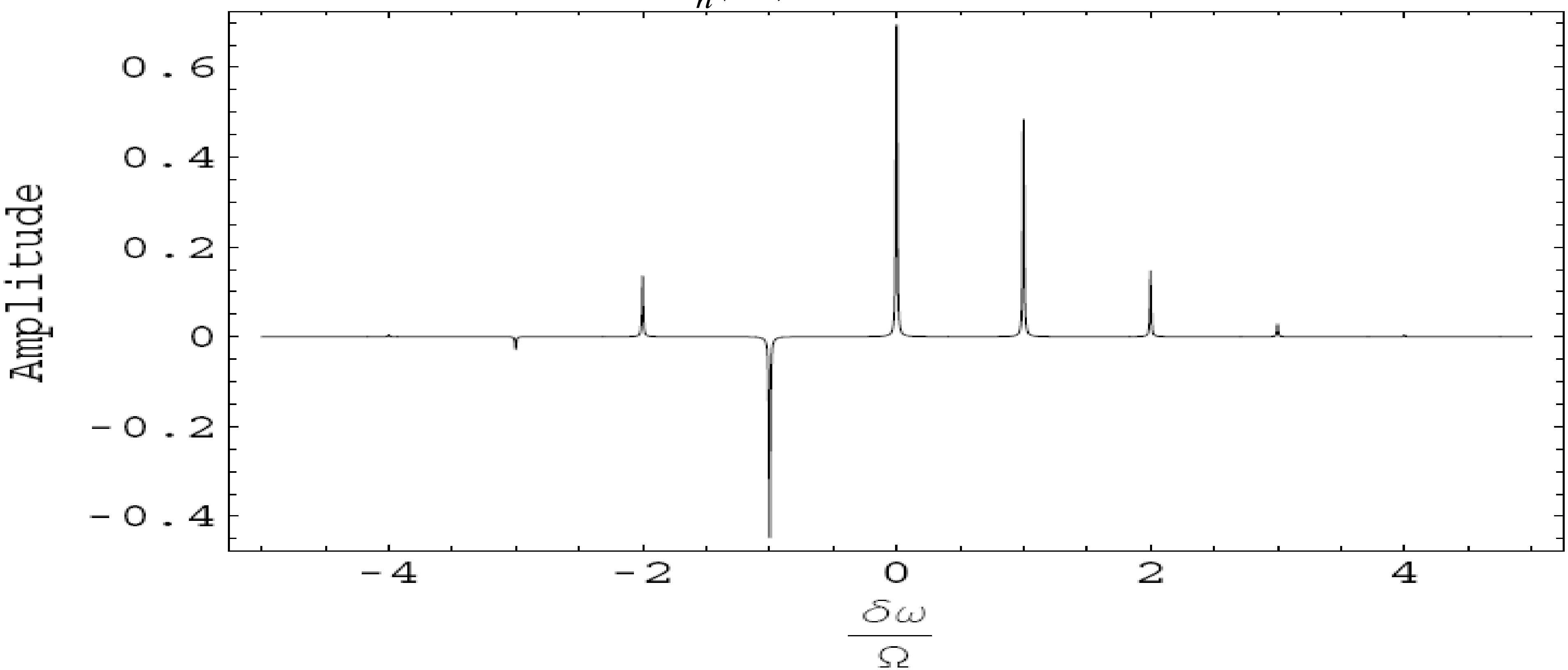
Electrooptic crystal:



Phase modulation

$$\vec{E}(t) = \frac{\vec{A}}{2} \sum_{n=-\infty}^{\infty} J_n(M) \exp\left(i(\omega + n\Omega)t\right) + c.c.$$

Bessel functions: $J_n(M)$



Modulation spectroscopy

Absorption of a side-band transfers phase modulation to amplitude modulation.